1936b, c, 1937) and stated that some subsidiary maxima were observed, but they did not analyse in detail the series of maxima arising from different N.

Recently we obtained the maxima in question with our newly-designed electron-diffraction camera of high resolution. Fig. 1(a) is a net pattern from a thin molybdenite crystal. Although the crystal was not rotated, a series of spots was recognized in the neighbourhood of the $(10\bar{1}0)$ spot due to a curving of the crystal. Fig. 1(b) is an enlarged picture of the marked part of Fig. 1(a). In this figure, a number of spots are clearly observed on both sides of the $(10\bar{1}3)$ spot. We have deduced from this figure, by applying the formula (4), that $Md = 210$ Å and $|V_{10\bar{1}3}| = 7{\cdot}2$ volts. The latter value is in good agreement with the theoretical value, 7·4 volts. The anomalous splitting observed in the peak $N = -2$ can be explained as an effect of the $(20\bar{2}6)$ reflexion which happens to occur simultaneously at this point. Thus, the pattern we obtained is without doubt that caused by the subsidiary maxima of the interference function in the dynamical theory of electron diffraction. We did not observe in our experiment the effect of the elastic bending of the crystal discussed by Blackman (1951).

The similar pattern was also observed independently by Hashimoto (1952) in the case of a thin MoO_3 crystal and by Ueda (1953) in the case of specially prepared colloidal gold.

Finally it is to be noted that Rees & Spink (1950) have also obtained similar subsidiary maxima in diffraction patterns from a ZnO crystallite. The maxima they obtained, however, are due to the small lateral extension of the crystallite, while the present ones are due to the small thickness of the crystalline film.

The cost of the diffraction camera used was met from a grant made by the Asahi Bunka Zaidan and the expense of this work was partly defrayed from the Scientific Research Grant of the Educational Ministry.

References

BLACKMAN, M. (1951). *Proc. Phys. Soc.* B, **64**, 625.
FINCH, G. I. & WILMAN, H. (1936a). *Nature, Lond.* **138**, 1010.
FINCH, G. I. & WILMAN, H. (1935b). *Proc. Roy. Soc.* A, **155**, 345.
FINCH, G. I. & WILMAN, H. (1936c). *Trans. Faraday Soc.* **32**, 1539.
FINCH, G. I. & WILMAN, H. (1937). *Ergebn. exakt. Naturw.* **16**, 353.
HASHIMOTO, H. (1952). Read at the Nagoya Meeting of the Electron Microscope Society, Japan, 10 October 1952.
KATO, N. & UYEDA, R. (1951). *Acta Cryst.* **4**, 229.
KOSSEL, W. & MÖLLENSTEDT, G. (1939). *Ann. Phys., Lpz.* (5), **36**, 113.
LIPSON, H. & STOKES, A. R. (1942). *Proc. Roy. Soc.* A, **181**, 101.
MACGILLAVRY, C. H. (1940). *Physica*, **7**, 333.
RAETHER, H. (1949). *Z. Phys.* **126**, 185.
REES, A. L. G. & SPINK, J. A. (1950). *Acta Cryst.* **3**, 316.
THOMSON, G. P. & COCHRANE, W. (1939). *Theory and Practice of Electron Diffraction*, p. 292. London: Macmillan.
UEDA, N. (1953). Read at the Osaka Meeting of the Physical Society of Japan, 8 April 1953.

Acta Cryst. (1954). **7**, 217

Subsidiary maxima of Kikuchi pattern.

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In the foregoing note (Uyeda, Ichinokawa & Fukano, 1954) we reported our observation on the subsidiary spots in a diffraction pattern, and showed that they can be explained in a similar way to those in a K.–M. pattern (Kossel & Möllenstedt, 1939) by the subsidiary maxima of the interference function in the dynamical theory. It is pointed out here that the subsidiary maxima of extinction fringes due to Bragg reflexion in electron micrographs of curved crystalline films (Heidenreich, 1949; Hashimoto, 1952) can be accounted for also by the same function.

The Kikuchi pattern has a different origin from the diffraction patterns mentioned above and, according to Kainuma (1954), it has an interference function as follows:

$$M\{1-\sin 2\pi M\sqrt{[l'^2+(2dq)^2]}/2\pi M\sqrt{[l'^2+(2dq)^2]}\}\,, \quad (1)$$

where M, l', d and q have the same meaning as in equation (3) of the previous note. Subsidiary maxima of this function appear at

$$l'_N = \pm\sqrt{[(n+\tfrac{3}{4})^2/M^2-(2dq)^2]}\,, \quad (2a)$$

where n is a positive integer and N is defined as $\pm n$ according to the double sign on the right-hand side. It must be mentioned here that the subsidiary maxima on the photographic plate coincide with (2) if the factor multiplying (1) is positive. When, however, the factor is negative, they appear at

$$l'_N = \pm\sqrt{[(n+\tfrac{1}{4})^2/M^2-(2dq)^2]}\,. \quad (2b)$$

The subsidiary maxima of Kikuchi patterns were reported by Heidenreich & Shockley (1948) to occur in a reflexion pattern from an aluminium single crystal. However, no observable subsidiary maxima are expected theoretically in reflexion patterns, and we suppose that the crystal studied by these authors might have been composed of several blocks with slightly different orientations. No subsidiary maxima have ever been observed, as far as we are aware, in parallel-beam transmission patterns. They are observable, though faint, in divergent-beam patterns taken by Kossel & Möllenstedt (1939) and by Hoerni (1950); but these authors apparently failed to take notice of the phenomenon.

We observed the maxima more clearly in divergent-beam diffraction patterns from a thin film of molybdenite; an example is reproduced in Fig. 1, where the $(20\bar{2}2)$-

Kikuchi pattern is accompanied by a few subsidiary maxima. This pattern is a superposition of a Kikuchi band and line, where the former predominates over the latter. Since the Kikuchi band has an asymmetric intensity near the position of the Bragg condition, the factor multiplying (1) is positive for positive N's and negative for negative N's. Therefore, the conditions (2*a*) and (2*b*) must be used respectively for positive and negative N's. Assuming these conditions, we analysed the pattern. The thickness of the film Md calculated from any successive subsidiary maxima was almost constant and the mean value turned out to be $Md = 520$ Å. This is in agreement with the thickness calculated from the $(21\bar{3}0)$-K.–M. pattern by the ordinary procedure. The values of $|V_{20\bar{2}2}|$ calculated from each maximum, even if showing rather large fluctuations, gave the mean value $|V_{20\bar{2}2}| = 1$ volt. This is in agreement with the theoretical value $|V_{20\bar{2}2}| = 1{\cdot}2$ volts. It should be noted here that if equation (3) of the previous paper is used instead of (1), $|V_{20\bar{2}2}|$ turns out to be imaginary if we assign to N the values 1, 2, 3 as given in the figure, and to be more than 10 volts if we assign the values 2, 3, 4. Similar agreement was also obtained in the case of some other indices. These facts may prove that we are here without doubt dealing with the subsidiary maxima of the interference function of Kikuchi patterns.

The reason that the subsidiary maxima are not observable in parallel-beam diffraction patterns may be as follows: The irradiated area of specimens in this case is much larger (usually $\gtrsim 100\,\mu$ diameter) than in the case of a divergent beam (usually $\lesssim 5\,\mu$ diameter). If crystals are thin enough to produce subsidiary maxima of resolvable angular widths, the curving of the crystal will, with larger irradiated area, inevitably disturb the appearance of Kikuchi pattern. It is worth mentioning that, contrary to the view held by early workers (Kikuchi, 1928), Kikuchi patterns appear even from quite thin crystals, if the irradiated area is reduced.

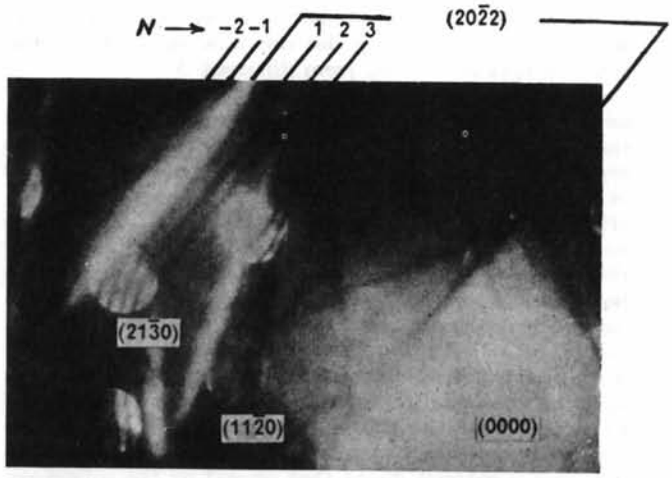

Fig. 1. Divergent-beam electron-diffraction pattern from a thin molybdenite film (520 Å).

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References

HASHIMOTO, H. (1952). Unpublished.
HEIDENREICH, R. D. (1949). *J. Appl. Phys.* **20**, 993.
HEIDENREICH, R. D. & SHOCKLEY, W. (1948). *Rep. Conf. 'Strength of Solids'*, p. 57. London: Physical Society.
HOERNI, J. (1950). *Helv. phys. Acta*, **23**, 587.
KAINUMA, Y. (1954). *Acta Cryst.* In the press.
KIKUCHI, S. (1928). *Jap. J. Phys.* **5**, 83.
KOSSEL, W. & MÖLLENSTEDT, G. (1939). *Ann. Phys., Lpz.* (5), **36**, 113.
UYEDA, R., ICHINOKAWA, T. & FUKANO, Y. (1954). *Acta Cryst.* **7**, 216.

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A dynamical theory of the simultaneous reflexion by two lattice planes. I. Simple representation of the dispersion surface.

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The dispersion surface in reciprocal space plays an important role in the dynamical theory of X-ray and electron diffraction. To represent this surface graphically, a cross-section with a plane is often used (e.g. Fues, 1939; Hoerni, 1950). In the present note, it will be shown that the theoretical formulation for the problem of simultaneous reflexion by two lattice planes, say with indices $h(h_1h_2h_3)$ and $h'(h'_1h'_2h'_3)$, can be simplified by choosing a particular plane for the cross-section.

The Schrödinger equation for the electron with an energy eE in the periodic potential $V(\mathbf{r}) = \sum_h V_h \exp 2\pi i \mathbf{b}_h\mathbf{r}$, where $\mathbf{b}_h$ is the vector representing the reciprocal-lattice point, is

$$\Delta\psi + \frac{8\pi^2 me}{h^2}(E+V(\mathbf{r}))\psi = 0\,. \quad (1)$$

Now ψ is considered to be composed of three waves corresponding to the indices 0, h and h', as follows:

$$\psi = u_0 \exp 2\pi i \mathbf{k}_0\mathbf{r} + u_h \exp 2\pi i \mathbf{k}_h\mathbf{r} + u_{h'} \exp 2\pi i \mathbf{k}_{h'}\mathbf{r}\ , \quad (2)$$

where $\mathbf{k}_h = \mathbf{k}_0+\mathbf{b}_h$, $\mathbf{k}_{h'} = \mathbf{k}_0+\mathbf{b}_{h'}$; and u_0, u_h and $u_{h'}$ are the amplitudes of the plane waves. The three functions $\varphi_m = \exp 2\pi i \mathbf{k}_m\mathbf{r}$ $(m = 0, h, h')$ are orthogonal to one another, as well as to the φ_m's of indices other than 0, h and h'. Here we replace these three functions by

$$\psi_0 = \varphi_0,\quad \psi_+ = \frac{1}{N}(\varphi_h + c\varphi_{h'}),\quad \psi_- = \frac{1}{N}(c\varphi_h - \varphi_{h'})\,, \quad (3)$$

where c is a constant, real and positive, and $N = \sqrt{(1+c^2)}$ is the normalizing factor. These functions are also orthogonal to one another, as well as to the φ_m's of indices other than 0, h, and h'. ψ is rewritten as